Engineering Notes

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Steady Transcritical Flow Past Slender Ships: A New Look

Allen Plotkin*
University of Maryland, College Park, Md.

Introduction

TUCK[1] obtained the solution for the steady potential flow past a slender ship in shallow water using the method of matched asymptotic expansions. Lea and Feldman[2] extended this solution into the transcritical (depth Froude number in the neighborhood of unity) range, but their solution is valid for the restrictive geometry of depth of $O(\epsilon^{2/3})$ where ϵ is the slenderness parameter. It is the purpose of this Note to demonstrate that the solution of Ref. 2 can be made applicable to the geometry considered by Tuck,[1] i.e., depth of $O(\epsilon)$, if it is recast with the slenderness parameter as the perturbation parameter.

Problem Formulation

Consider the steady potential flow of a uniform stream of speed U past a rigid slender ship in shallow water of depth ϵh. A Cartesian coordinate system is introduced with its origin in the undisturbed free surface with x in the streamwise direction and z positive upwards. The ship hull is given by $z=\epsilon f(x,y)$ and the free surface by $z=\eta(x,y)$ where ϵ is a measure of the ratio of beam or draft to ship length (which is $O(1)$).

The velocity is taken as $U\nabla(x+\phi)$ where ϕ is the perturbation potential and satisfies Laplace's equation. Kinematic boundary conditions on the free surface and bottom and a dynamic condition of constant pressure on the free surface are satisfied by ϕ. The solution technique of Lea and Feldman[2] will be used with the slenderness parameter (instead of the depth) as the perturbation parameter. The ordering of terms is motivated by a study of the corresponding transonic airfoil problem (see Ashley and Landahl[3]).

Since ϵ is an appropriate scale in the vertical direction, the coordinate $Z=z/\epsilon$ will be used. The Froude number F is given by

$$F^2 = U^2(gh\epsilon)^{-1} = 1+\epsilon^{2/3}K \tag{1}$$

where g is the gravitational acceleration and K is a constant.

Outer Problem

The appropriate far field transverse variable is

$$\bar{y}=\epsilon^{1/3}y \tag{2}$$

The perturbation potential and free-surface elevation are expanded in perturbation series as

$$\phi=\sum_{1}\epsilon^{2n/3}\phi^{(n)}(x,\bar{y},Z) \tag{3a}$$

and

$$\eta=\epsilon\sum_{1}\epsilon^{2n/3}\eta^{(n)}(x,\bar{y}) \tag{3b}$$

Following Lea and Feldman,[2] the differential equation for $\phi^{(1)}(x,\bar{y})$ is obtained as

$$\phi^{(1)}_{xx}[3\phi^{(1)}_x+K]-\phi^{(1)}_{\bar{y}\bar{y}}=0 \tag{4}$$

The inner expansion of this outer solution is

$$\phi^{(1)}\sim\epsilon^{2/3}[\phi^{(1)}(x,0)+|\bar{y}|\phi_{\bar{y}}^{(1)}(x,0+)+\ldots] \tag{5}$$

Inner Problem

The appropriate near field transverse variable is

$$Y=y/\epsilon \tag{6}$$

The velocity potential has the expansion

$$\phi=\epsilon^{2/3}\Phi^{(1)}+\epsilon^2\Phi^{(2)}+\ldots \tag{7}$$

where $\Phi^{(1)}$ is a function of x only and $\Phi^{(2)}$ satisfies Laplace's equation in the crossflow plane with a nonzero normal derivative on the hull. It has an outer expansion of

$$\phi\sim\epsilon^{2/3}[\Phi^{(1)}(x)+US'(x)|\bar{y}|/2h]+\ldots \tag{8}$$

where $\epsilon^2 S(x)$ is the wetted cross-sectional area of the hull. Matching of the inner and outer expansions (see Van Dyke[4]) yields

$$\Phi^{(1)}(x)=\phi^{(1)}(x,0) \tag{9a}$$

and

$$\phi_{\bar{y}}^{(1)}(x,0+)=US'(x)/2h \tag{9b}$$

To obtain the lowest-order pressure distribution, vertical force, and pitching moment on the ship we need to solve the differential equation (4) with the hull boundary condition (9b). This is the identical problem solved by Lea and Feldman[2] and results are presented in their paper. In conclusion, their solution for the steady transcritical flow past a slender ship in shallow water has been shown to be applicable when the beam, draft, and water depth are of comparable magnitude.

Acknowledgment

This research was supported by the National Science Foundation under Grant No. ENG 74-20573.

References

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[3] Ashley, H. and Landahl, M.T., *Aerodynamics of Wings and Bodies*, Addison-Wesley, Reading, Mass., 1965, Ch. 12.

[4] Van Dyke, M., *Perturbation Methods in Fluid Mechanics*, Parabolic Press, Stanford, Calif., 1975, Ch. V.

Received Dec. 13, 1976.
Index category: Hydrodynamics.

*Associate Professor of Aerospace Engineering. Member AIAA.